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R & D NOTES

Turbulent Non-Newtonian Transport in a Circular Tube

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This work is concerned with turbulent heat or mass transfer in a circular tube to nonelastic fluids whose rheological behavior can be approximated by the power law model expressed as

$$\tau = K\left(\frac{du}{dy}\right)^n \qquad (1)$$

For the important case of large Prandtl numbers, an analytical expression for the heat transfer coefficient is derived. The asymptotic results obtained depend on the limiting behavior of the eddy diffusivity near the wall. An expression for the eddy diffusivity variation in the wall region derived by Notter and Sleicher (1971) was used in this work. This expression was derived from con-

Table 1. Results of the Least-Squares Analysis of the Heat Transfer Data

No. of data points*	B_1	B_2	B_3	Mean deviation†	Standard deviation**
106	19.122	12.743	−21.638	14.86%	20.60%
88	13.968	37.840	−31.949	12.54%	19.04%

* Runs made with 88 data points correspond to Prandtl numbers larger than 10. Runs made with 106 points correspond to the complete Prandtl number range.

† Mean deviation $= \frac{1}{N} \sum_{i=1}^{N} \left| \frac{St_{pred} - St_{exp}}{St_{exp}} \right|$.

** Standard deviation $= \left[\frac{1}{N-3} \sum_{i=1}^{N} \left(\frac{St_{pred} - St_{exp}}{St_{exp}} \right)^2 \right]^{1/2}$.

siderations of heat transfer at large Prandtl number and mass transfer at large Schmidt number. The resulting asymptotic formula for the non-Newtonian heat transfer coefficient contains no adjustable parameters and was found to fit the available data as well as the semitheoretical relationship proposed by Metzner and Friend (1959), which contained one adjustable parameter.

DEVELOPMENT OF LARGE PRANDTL NUMBER ANALYTICAL RESULTS

The wall boundary condition considered is that of a uniform wall heat flux. For this condition, the fully developed temperature profiles are described by $(\partial t/\partial x) =$ a constant, and the differential equation describing the temperature distribution may be written in dimensionless form, neglecting viscous dissipation, as

$$\frac{-4u^+(r^+)}{PrRe} = \frac{1}{r^+}\frac{d}{dr^+}\left[r^+\left(\frac{1}{Pr} + \epsilon^+\right)\frac{dT^+}{dr^+}\right] \tag{2}$$

The viscosity used in Pr, Re, and ϵ^+ is defined in terms of the wall shear stress. Equation (2) may be integrated twice and the resulting temperature distribution substituted into the integral defining the bulk temperature to give, after the order of integration is changed, an expression for the Stanton number:

$$\frac{\sqrt{f/2}}{St} = \frac{4}{Re^2}\int_0^{R^+} \frac{\left\{\int_{y^+}^{R^+} 2u^+(z^+)\left[1 - \frac{z^+}{R^+}\right] dz^+\right\}^2 dy^+}{\left(1 - \frac{y^+}{R^+}\right)\left(\frac{1}{Pr} + \epsilon^+\right)} \tag{3}$$

Since most Prandtl numbers of interest in non-Newtonian systems will be large, we are interested in an asymptotic representation of the integral in Equation (3) for the limit of large Pr. The major contribution to the integral in this case comes from the wall region. Therefore, we need to have an expression for the eddy diffusivity ϵ^+ which is valid in this region. Since most evidence is for the Taylor series approximation for ϵ^+ in the wall region beginning with y^{+3} (Harriott and Hamilton, 1965; Hubbard and Lightfoot, 1966; Notter and Sleicher, 1971), we have considered the case where ϵ^+ is given by Equation (4):

$$\epsilon^+ = K_3 y^{+3} + K_4 y^{+4} + K_5 y^{+5} + \ldots \tag{4}$$

Hanna and Sandall (1972) have asymptotically evaluated the integral in Equation (3) for large Pr with ϵ^+ given by Equation (4). They found

$$\frac{\sqrt{f/2}}{St} = B_1 Pr^{2/3} + B_2 Pr^{1/3} + B_3 \ln Pr + O(1) \tag{5}$$

The coefficients B_1, B_2, and B_3, which are given by Hanna and Sandall, are known functions of the coefficients in the eddy diffusivity expression and the Reynolds number. The Reynolds number dependence of the coefficients, however, is extremely weak, and for practical purposes the coefficients may be taken as constants.

COMPARISON WITH EXPERIMENTAL DATA

The data used by Metzner and Friend to develop their correlation were used to test the ability of Equation (5) to fit non-Newtonian heat transfer data. These data cover the following ranges for the variables:

Flow behavior index, n: 0.39 to 1.00
Prandtl number, Pr: 1.88 to 264
Reynolds number, Re: 1 070 to 141 000

A nonlinear least-squares fitting procedure was used to determine the constants B_1, B_2, and B_3. The results of this data are shown in Table 1. The Stanton number expression resulting from this data fitting is

$$\frac{\sqrt{f/2}}{St} = 19.12 Pr^{2/3} + 12.74 Pr^{1/3} - 21.64 \ln Pr \tag{6}$$

The agreement with the data was found to be somewhat better than the equation of Metzner and Friend:

$$St = \frac{f/2}{1.18 + \sqrt{f/2}\,\{11.8(Pr - 1)Pr^{-1/3}\}} \tag{7}$$

The mean deviation was 17.6% and the standard deviation 23.5% when the data (106 data points) are compared to the Metzner and Friend correlation, Equation (7).

USE OF THE NOTTER AND SLEICHER EDDY DIFFUSIVITY

A recent article by Notter and Sleicher (1971) proposed the following expression for the eddy diffusivity in the wall region:

$$y^+ < 45, \quad \epsilon^+ = \frac{0.00091 y^{+3}}{(1 + 0.0067 y^{+2})^{1/2}} \tag{8}$$

This expression accounts for ϵ^+ varying as y^{+3} near the wall and as y^{+2} at some distance from the wall. The care with which this expression was derived and compared with Newtonian heat and mass transfer data gives confidence in its validity.

If it is supposed that Equation (8) also applies to non-Newtonian fluids, then this relationship could be used to integrate Equation (3). In this case, since Equation (8) is valid only for $0 < y^+ < 45$, the integral in Equation (3) must be evaluated in two parts:

$$\frac{\sqrt{f/2}}{St} = \int_0^{45} \frac{G(y^+)}{\left(\frac{1}{Pr} + \epsilon^+\right)} dy^+ + \int_{45}^{R^+} \frac{G(y^+)}{\left(\frac{1}{Pr} + \epsilon^+\right)} dy^+ \tag{9}$$

In the limit of large Prandtl numbers, the contribution of the second term in Equation (9) is very small compared to the first term and is usually neglected. However, the heat transfer data considered here correspond to a

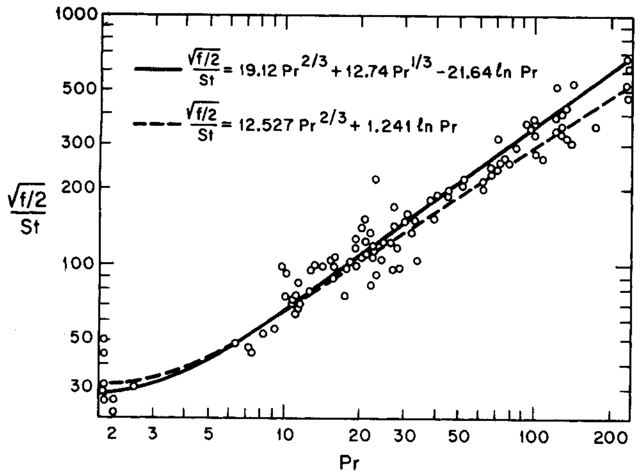

Fig. 1. Comparison of experimental non-Newtonian heat transfer data with theoretical formulas. Data taken from Farmer (1960), Friend (1959), Haines (1957), Raniere (1957).

TABLE 2. COMPARISON OF ANALYTICAL APPROXIMATION WITH EXACT NUMERICAL INTEGRATION OF EQUATION (10) BY USING NOTTER AND SLEICHER EQUATION FOR ϵ^+

Pr	Analytical formula (Equation 11)	Numerical integration
1,000	1261.3	1262.9
500	796.88	798.52
100	275.61	277.39
50	174.88	176.67
10	61.00	62.86
1	12.53	14.70

range of Prandtl numbers between 1.88 and 260, where the turbulent core contribution may be of some importance. Consequently, the second term was retained.

For large Pr, the first integral in Equation (9) may be approximated very closely by

$$I_1 = \int_0^{45} \frac{dy^+}{\left(\frac{1}{Pr} + \epsilon^+\right)} \quad (10)$$

By using the technique of Hanna and Sandall (1972), the integral in Equation (10) may be evaluated asymptotically for large Pr with ϵ^+ given by Equation (8) to yield

$$I_1 = 12.527Pr^{2/3} + 1.241\ln Pr \quad (11)$$

Table 2 compares the analytical approximation to I_1 given by Equation (11) with an exact numerical integration of Equation (10) with ϵ^+ given by Equation (8). The agreement is seen to be very close at large Pr, and the approximation holds relatively well even at Prandtl numbers as low as 1. At $Pr = 1$, Equation (11) is off by 14.8% compared to the numerical integration of Equation (10), and this deviation decreases rapidly as Pr increases, giving a deviation of 0.13% at $Pr = 1{,}000$.

The second term in Equation (9), representing the contribution of the turbulent core, was evaluated by using a Prandtl-Taylor type of analogy and the velocity distribution of Clapp (1961) to give

$$I_2 = \frac{2.78}{n}\ln\left(\frac{Re\sqrt{f/2}}{90}\right) \quad (12)$$

Substitution of Equations (11) and (12) into Equation (9) gives an expression for the Stanton number:

$$\frac{\sqrt{f/2}}{St} = 12.527Pr^{2/3} + 1.241\ln Pr + \frac{2.78}{n}\ln\left(\frac{Re\sqrt{f/2}}{90}\right) \quad (13)$$

Equation (13) contains no free parameters and, when compared to the non-Newtonian heat transfer data, gives a mean deviation of 16.6% and a standard deviation of 23.4%. These results, though not quite as good as those given by the fit of the data, are still as good as those of Metzner and Friend. Moreover, Equation (13) contains no free parameters, while Equation (6) contains three and the Metzner and Friend correlation has one.

The heat transfer data are compared to Equations (6) and (13) in Figure 1.

NOTATION

B_1, B_2, B_3 = constants in Equation (5)
C_p = heat capacity, cal/g °K
f = friction factor = $2\tau w/\rho u_b^2$

$$G(y^+) = \frac{4}{Re^2}(1 - y^+/R^+)\left\{\int_{y^+}^{R^+} 2u^-(z^+)\left[1 - \frac{z^+}{R^+}\right]dz^+\right\}^2$$

$$I_1 = \int_0^{45} \frac{G(y^+)}{\left(\frac{1}{Pr} + \epsilon^+\right)} dy^+$$

$$I_2 = \int_{45}^{R^+} \frac{G(y^+)}{\left(\frac{1}{Pr} + \epsilon^+\right)} dy^+$$

k = thermal conductivity, cal/cm °K s
K = flow consistency index, g/cm s^{2-n}
K_1, K_2, K_3 = constants in Equation (4)
n = flow behavior index
$O(\,)$ = order symbol, $A = O(Pr^a) \Longrightarrow A\alpha Pr^a$, for $Pr \rightarrow \infty$
Pr = Prandtl number = ν^*/α
q = wall heat flux, cal/cm^2
r = radial coordinate, cm
r^+ = dimensionless radial coordinate = ru^*/ν^*
R = tube radius
R^+ = dimensionless tube radius = Ru^*/ν^*
Re = Reynolds number = $2u_bR/\nu^*$
St = Stanton number = $h/\rho C_p u_b$
T = temperature, °K
T_w = wall temperature, °K
T^+ = dimensionless temperature = $k(T_w - T)u^{*2-n/n}/q(K/\rho)^{1/n}$
u = velocity, cm/s
u_b = bulk velocity = $\frac{2}{R^2}\int_0^R urd\nu$, cm/s
u^+ = dimensionless velocity = u/u^*
u^* = friction velocity = $\sqrt{f/2}\,u_b$, cm/s
x = axial coordinate, cm
x^+ = dimensionless axial coordinate = xu^*/ν^*
y = distance from wall = $R - r$, cm
y^+ = dimensionless distance from wall = yu^*/ν^*
z^+ = dummy variable for y^+

Greek Letters

α = thermal diffusivity = $k/\rho C_p$, cm^2/s
ϵ^+ = dimensionless eddy diffusivity for heat = ϵ/ν^*
ν^* = effective kinematic viscosity = $(K/\rho)^{1/n}/u^*\,2 - 2n/n$

ρ = density, g/cm³

τ = shear stress, g/cm s²

τ_w = wall shear stress, g/cm s²

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On Reversible Adsorption of Hydrosols and Repeptization

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In a previous paper (Ruckenstein and Prieve, 1976) the reversible deposition of particles from a moving fluid was treated by lumping the effect of the interaction forces between particles and the deposition surface in a boundary condition to the usual convective-diffusion equation. The boundary condition was obtained by analyzing the flux of particles through a very thin layer (~ 100Å) next to the collector's surface. Within this interaction force boundary layer, a quasi steady state was assumed up to a point between the maximum and primary minimum (in the profile of the potential energy of interaction), with quasi equilibrium assumed between that point and the wall (a region in which most of the accumulation is expected to occur). In what follows, this is called approximation A. The goals of this note are: (1) to compare approximation A with approximation B, which assumes quasi steady state over the entire thickness of the interaction force boundary layer, and (2) to extend the analysis to flocculation and repeptization of hydrosols.

In one dimension the equation of continuity for the particles within the thin layer is

$$\frac{\partial c}{\partial t} + \frac{\partial J}{\partial h} = 0$$

where the nomenclature is that of Ruckenstein and Prieve (1976) and J is given by Equation (6) of that same paper. To prevent particles from penetrating the wall at $h = 0$ and to match the concentration at $h = \delta_0$ with that outside the interaction force boundary layer, the following boundary conditions must be imposed:

$$J = 0 \quad \text{at} \quad h = 0$$
$$c = c_i \quad \text{at} \quad h = \delta_0$$

where δ_0 is the thickness of the interaction forces boundary layer. The solution of this unsteady problem would allow the accumulation of particles near the surface to be evaluated. More details will be found in Prieve and Ruckenstein (to be published).

When the interaction forces establish an energy barrier that reduces adsorption and desorption rates significantly, particles around the primary minimum ($0 \leq h \leq \delta_2$) have time to achieve quasi equilibrium before their population changes. Then, assuming a quasi steady state for the remainder of the region, we have (alternative A)

$$J = 0 \quad \text{for} \quad 0 \leq h \leq \delta_2$$
$$\partial J/\partial h = 0 \quad \text{for} \quad \delta_2 < h \leq \delta_0$$

where δ_2 must be chosen strictly less than $h_{\max}$. This position ($h_{\max}$), corresponding to a maximum in the interaction potential, is unstable; therefore, it is not reasonable to assume quasi equilibrium there.

Note that the boundary condition $J = 0$ at $h = 0$ is automatically satisfied. Using the other boundary condition and Equation (6), we previously obtained Equation (11)

$$-J = \frac{c_i \left(\int_0^{\delta_2} e^{-\phi/kT}\, ds \right) - n_2}{\left(\int_0^{\delta_2} e^{-\phi/kT}\, ds \right) \left(\int_{\delta_2}^{\delta_0} D^{-1}\, e^{\phi/kT}\, ds \right)} \quad \text{(A)}$$

for the net adsorption rate, where n_2 is the number of adsorbed particles per unit area, or

$$n_2 = \int_0^{\delta_2} c\, ds$$

One may be tempted to replace $h = \delta_0$ by $h = \infty$. However, this causes the integrals to diverge (Prieve and Ruckenstein, 1976) because of the way the diffusion coefficient (or particle mobility) tends to its bulk value. Prieve and Ruckenstein (1976) have shown that for a low potential barrier the integral depends on the value of δ_0. Because it is not possible to define a precise value of δ_0, the lumping of the effect of the interaction forces in a boundary condition is meaningless in this case. If the potential barrier is high enough, the integral is practically independent of δ_0,